On the Road to God: Einstein's Imaginary Journey Around the Universe

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A revived figure of teenage Albert Einstein is confronted at the threshold of the second millennium with a problem of a graphical likeness of the Lambert W function-based cosmological equations to a logarithmic-exponential discontinuous function that he has just plotted. To puzzle the issue out his mind is put onto an imaginary mathematical-philosophical-physical trail ride around the Universe. Euler's identity that he invited to ride pillion on his Pegasus of Imagination spanks the cavalier's horse to carry him on wings of Leibniz' zero-nothingness unity-infiniteness array towards a non-Euclidean landscape of novel cosmological equations – which describe behaviour of principal cosmological parameters as an exclusive function of the scale factor. Zero, unity, π, and Euler's number become in the rider's eyes a quaternion of linesmen – and the imaginary unit the one who as a chief referee opens and controls on the space-time playground a collective mass-energy game. And while the young cavalier observes from a horseback of his mount that within the playground expansion the cosmic match is running into the order of timelessness and spacelessness the Universe with all of its ever existing forms and events, thoughts and memories, unveils to him its all-encompassing face of eternity and immortality attributable to God.

Keywords: Albert Einstein, Euler's identity, God, imagination, Lambert W function, scale factor, spacetime, thought experiment, Universe

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Introduction

"...nature represents its simplicity in various physical systems..."
(Nojiri et al., 2017)

The famous portrait of teenage Albert Einstein taken in a Munich photo studio presents a high school student sitting on the top of painted seashore highlands (Fig. 1). His inner imaginative-like gaze of a dreamy mind seems to glimpse an infinitive scheme of sailing waves of time and space over the imaginary landscape. In two years later, being 16-years-old guy, his thoughts of the physical world will give birth to a very curious idea: *"What it would be like to pursue a beam of light?"* (Norton, 2013). By this question his most famous thought experiment (Gedankenexperiment) is ready to open the era of relativistic physics. But what kind of the thought experiment would have been performed by him as an abecedarian of the 21st century cosmology virtual class? Would have been his mind challenged by a quizzical daydream of a trail ride around the expanding universe? This is an oddish idea, indeed, but it came into the author's mind like a prepossessing wavy sparkplug above a return train of thought heading times of Einstein's youth. At an early point its terminal station was on the air; however, it was just a double reason and challenge to get on.

1. The Lambert W Function and Einstein's Novel Thought Experiment

In early 2020, the Lambert W function (Corless et al., 1996) being already well established in its generalized form in the class of relativistic physics (Scott et al., 2006; Mező and Baricz, 2017) is introduced as a potential *"...newcomer in the cosmology class"* (Saha & Bamba, 2020). The same year, a novel model of regular black holes highlights this special function along with exponentials for its *"...somewhat more elegant..."* quality when compared to *"...rather messy cubic and quartic polynomial equations"* (Simpson & Visser, 2020). Two years later a comprehensive treatise on the Lambert W function is published (Mező, 2022). The function which *"...seems to be ubiquitous in nature..."* (Corless et al., 1996) is invaluable in its ability of solving transcendental equations variable of which is found simultaneously outside and inside logarithms, like: $x = \ln(3x)$. Or outside and inside exponentials, like $y = xe^x$; the famous equation the first references and studies of which were made by Johann Heinrich Lambert (1758) and Leonhard Euler (1783), 18th century Swiss mathematicians (Brito et al., 2008)).

Thereupon, at the threshold of the second millennium a revived figure of teenage Albert Einstein presents in the cosmology class an iterative solution[1] of the transcendental equation $x = [\log(x).\ln(x)]^3$. In order to set up its first approximation he makes a plot[2] of a logarithmic-exponential discontinuous function $\sqrt[3]{x}\,/\ln(x)$ against $\log(x)$. The resulting graph (Fig. 2) immediately strikes his eye as it looks as if in terms of reflection symmetry two graphs (Fig. 3) of the Lambert W function-based cosmological equations for the deceleration parameter q, and for the energy density ρ, respectively (Saha and Bamba, 2020) were put together. The similarity provokes him to wondering about an eventual significancy of the examined function $\sqrt[3]{x}\,/\ln(x)$ to cosmology. His query is: "What if it is descried there in the variable x a spatiotemporal dynamic factor which makes the function numerator $\sqrt[3]{x}$ attributable to the non-Euclidean spatiotemporal geometry and concurrently the function denominator $ln(x)$ attributable to the maximum possible entropy referred to a measure of information uncertainty (Silagadze, 2010) of the mass-energy system?" To examine the idea he modifies the function $\sqrt[3]{x}\,/\ln(x)$ in two different ways; separately for $log(x) > 0$ and for

$log(x) < 0$. The 'plus way' takes him to a general scheme of the deceleration parameter equation; while the 'minus way' to a general scheme of the equation for the squared Hubble parameter (its square represents an equivalent to the energy density). Within the 'minus way' he makes use of a fundamental 'trick' of absolute logarithmic values $|log(x) < 0| = log(1/x)$; and within both ways he takes in a novel dynamic spatiotemporal factor e^{φ}.

$$(e^{\varphi} - 1)\,[\sqrt[3]{(1/x)} \,/\, \ln(x)] \xleftarrow{\log(x)<0} \sqrt[3]{x} \,/\, \ln(x) \xrightarrow{\log(x)>0} e^{\varphi} - [\sqrt[3]{x} \,/\, \ln(x)]$$

Both spatiotemporal dynamic factors x and e^{φ} call for their specification. Juvenile Einstein is aware of invalidity (respectively instability) of 'his' initial model of the static universe as it implies zero time derivatives for the scale factor $\dot{a} = \ddot{a} = 0$. So his mind is switched to a dynamic model of the expanding universe – which makes him put a trick question: "In case I will parametrize the spatiotemporal factor x just through the scale factor what it would be like to use a combined π-shaped and a-shaped pair of reins to ride Pegasus of Imagination around the expanding universe?" Instead a reply he grasps a-shaped rein in his left and an π-shaped rein in his right hand. And as he takes $log(x)$ to keep them in balance he gets the formula $x = 10^{\pi a}$. By analogy he takes the scale factor to parametrize both the spatiotemporal dynamic factor e^{φ} (that he decides to be defined for $(1; e>$ and to be called the 'perpetual factor') and its variable φ (that is therefore defined for $(0; 1>$ and called by him the 'dark factor'). So he grasps φ_0 as a reference constant for a present epoch (its value will be specified later on) in his left hand and the scale factor a again in his right hand. And when he takes φ to keep this pair of reins in balance he gets the formula $\varphi = \varphi_0 a$; and accordingly $e^{\varphi} = e^{\varphi_0 a}$.

2. Euler's Identity and Cosmological Equations

The parametrized factors $10^{\pi a}$ and $e^{\varphi_0 a}$ as they were received by juvenile Einstein operate explicitly with transcendental numbers of the eminent importance in mathematics – π and e, respectively; and implicitly with two more eminent numbers, 0 and 1. The fact navigates the explorer's train of thought towards the idea that spacetime could be under the continuous π - e - zero – unity orchestrated action based on a scheme of Euler's identity – a specific case of Euler's formula when its variable x is put equal to π:

$$e^{ix} = \cos(x) + i\sin(x) \xrightarrow{for\ x=\pi} e^{i\pi} + 1 = 0$$

Since Euler's identity formalizes growth as a circular motion which cannot be escaped within the area of complex numbers any way the trail rider deliberates that its scheme could explain why there is no escape to him beyond the universe. And why the identity could symbolize a wholeness of the universe existence. But what makes him wonder is the fact that the imaginary unit i holds in the identity simultaneously a role of its animator and of the keeper of its steady and irrefrangibly universal validity. Which evokes in his head the idea that the universe existence depends on the imaginary unit inherency – and that a 'super' transcendental number $e^{\pi} + 1$ could mark the maximum scale factor of the universe. Defining the dark factor on grounds of this notion as $\varphi = a/(e^{\pi} + 1)$ enables him to refine general schemes of the deceleration parameter and the Hubble parameter equations (while using the formula $x = 10^{\pi a}$ and an empirical constant[3] of correlation 10^7 $[km^2 \cdot s^{-2} \cdot Mpc^{-2}]$ as well) into their definite forms presented below. Behaviour of both parameters these equations describe for a given range of the scale factor[4] is presented in Fig. 4. The Hubble parameter equation is viewed by him as a potential arbiter of the 'Hubble tension' disputes (Di Valentino et al., 2021; Freedman, 2021; Krishnan et al., 2021; Rameez & Sarkar, 2021).

$$q = e^{a/(e^{\pi}+1)} - [\sqrt[3]{10^{\pi a}} / \ln(10^{\pi a})]$$

$$H = \left\{(e^{a/(e^{\pi}+1)} - 1)[\sqrt[3]{1/10^{\pi a}} / \ln(10^{\pi a})] \cdot 10^{7}\right\}^{1/2} \ [km \cdot s^{-1} \cdot Mpc^{-1}]$$

At the next stop the juvenile cosmic rider meets a model of the perfect cosmic fluid that allows description of the universe by an effective equation of state w_{eff}; via the relation of the fluid pressure P to the energy density ρ. Both of these physical quantities belong to key elements of the Friedmann equations (represented by the acceleration equation and by the Friedmann equation) which are derived from Einstein field equations of General relativity. Putting their simplified forms (the metric with $c = 1$ for the speed of light is considered for commoving coordinates (Saha & Bamba, 2020)) together with the time derivative of the Hubble parameter enables young Einstein to formulate a simple effective equation of state: $w_{eff} = (2q-1)/3$. The deceleration parameter as its exclusive variable is represented by the equation that was formulated by him above.

$$\dot{H}/H^2 = -(1+q) \qquad 3H^2 = 8\pi G\rho$$
$$(time\ derivative\ of\ the\ Hubble\ parameter) \qquad (simplified\ Friedmann\ equation)$$

$$\dot{H} + H^2 = -(4\pi G/3)(\rho + 3P)$$
$$(simplified\ acceleration\ equation)$$

$$\dot{H} = -H^2 - qH^2 = -(4\pi G/3)(\rho + 3P) - H^2$$

$$q(8\pi G\rho/3) = (4\pi G/3)(\rho + 3P) \ \longrightarrow \ 2q\rho = \rho + 3P \ \longrightarrow \ P/\rho = (2q-1)/3$$

$$w_{eff} = (2q-1)/3 \ \longrightarrow \ w_{eff} = 2/3\,[e^{a/(e^{\pi}+1)} - (\sqrt[3]{10^{\pi a}} / \ln(10^{\pi a}))] - 1/3$$

Having been originally formulated on grounds of the Lambert W function[5] (Saha and Bamba, 2020) and suggested then *"...to serve as a unification of dark matter and dark energy..."* (Al Mamon & Saha, 2021) an effective equation of state w_{eff} may be seen as the rationale for a deliberation on the novel dynamical Lambert W Dark Energy model instead the Lambda cold dark matter (ΛCDM) model (Banerjee et al., 2021). Yet, some of the modified gravity theories (Nojiri et al., 2017) in which the purely geometric nature of dark energy is considered (Mandal et al., 2020) operate just with an extension of the ΛCDM model. And as regards a position of young Einstein he is convinced that the ΛCDM model authenticity and simplicity are like two sides of the same universal coin fineness of which is going to be argued in the following text.

3. The Scale Factor As an Exclusive Cosmological Variable

A call for a precise measurement of both the Hubble and the deceleration parameters (Sandage, 1970) led to proposition additional cosmological parameters and to formulation novel cosmological models (Neben & Turner, 2013). While they have been thought to improve understanding the universe expansion it has been comparison statistics (Davis et al., 2007) which argued in favour of *"...models that give a good fit with fewer parameters...,"* with reference to that *"...the preferred cosmological model is the flat cosmological constant model, where the expansion history of the universe can be adequately described with only*

one free parameter describing the energy content of the universe..." Ultimately, this model was found to remain the simplest and the best *"...to explain the current data..."* while giving *"...no reason to prefer any more complex model over the concordance cosmology, the flat cosmological constant."* Hence, the ΛCDM model authenticity should be judged rather from *"...either new physics or unknown systematics..."* perspectives – as it was proposed for a resolution of the tension (D'Arcy Kenworthy et al., 2019).

The latter perspective of the yet *"...unknown systematics..."* is favoured by juvenile Einstein since it points to the fact that behaviour of cosmological parameters as it is described by the equations proposed above conforms an accelerated expansion of the universe (Riess et al., 1998) and supports an authenticity of the ΛCDM model with its hypothetical phantom energy action that would lead to a Big Rip. In the explorer's opinion this event is about to happen when the scale factor of the universe meets the constant $e^{\pi}+1$. Both the maximum radius r and the maximum age A of the universe may be calculated then according the following equations (the factors $1\ Mpc \approx 3.0857 \cdot 10^{19}\ [km]$ and $1\ year \approx 31\ 556\ 952\ [s]$ are considered for a conversion of the Hubble parameter $H_{\tau}\ [s^{-1}]$ to $H\ [km \cdot s^{-1} \cdot Mpc^{-1}]$).

$$r = ac/H_{\tau} \approx (3.0857ac/H) \cdot 10^{19}\ \ [km]$$

$$A = r/c = a/H_{\tau} \approx (3.0857a/\ 31556952\ H) \cdot 10^{19} \approx 0.97782\ (a/H) \cdot 10^{12}\ [years]$$

According these equations both the radius and the age of the universe are proportional to the scale factor and reciprocal to the Hubble parameter. However, since the scale factor is the only variable of the Hubble parameter equation (see a previous section) both the radius and the age of the universe have actually become exclusive functions of the scale factor.[6]

4. The Time Dilation Factor and the Fate of the Universe

The young adventurer on his imaginary trail ride around the expanding universe is heading now a hypothesized sphere of the time dilation. He views it as a ratio of the 'actual time flow velocity' v_a to the 'initial time flow velocity' v_i that was effective in the earliest flash of the universe. Apart from an elusiveness of their absolute values he finds their ratio to be equal to a ratio of two specific time flow gradients defined on grounds of Euler's number and with regard to Euler's identity. According his assumption the 'actual time flow velocity' v_a corresponds to the 'perpetual factor time flow gradient' $e - e^{a/(e^{\pi}+1)}$ and accordingly the 'initial time flow velocity' v_i to the 'singularity time flow gradient' $e - e^0$. Which enables him to formulate a dimensionless equation for the time dilation factor $\Delta v_{d\tau}$; along with an equation for its impact on the age of the universe $A_{d\tau} = A\ \Delta v_{d\tau}\ [years]$. For a present epoch $(a_0 = 1)$ the time dilation factor is calculated by him to $\Delta v_{d\tau(a_0)} \approx 1.00834$.

$$\Delta v_{d\tau} = 1/\left(\sqrt[3]{v_a}\,/\sqrt[3]{v_i}\right) = \sqrt[3]{e - e^0}\,/\sqrt[3]{e - e^{a/(e^{\pi}+1)}} = \sqrt[3]{e-1}\,/\sqrt[3]{e - e^{a/(e^{\pi}+1)}}$$

Juvenile Einstein is realizing now that a waveform of the time dilation factor as a function of the scale factor (Fig. 5) is strikingly similar to that of the Lorentz factor as a function of the relative velocity (Fig. 6). And that the similarity is of the physical nature. Unity represents a ground level from which both the time dilation factor and the Lorenz factor very gradually rise up at first; while the late accelerating phase corresponds to an uncontrollable expansion of the universe – respectively to velocity that is nearing the speed of light. When its limit

is met the time flow velocity is frozen up and the universe – since there is no time nor light anymore – should vanish away. A similar scenario emerges from a novel equation of state (Saha & Bamba, 2020) based on incorporation the Lambert W function; as both scenarios outline a cosmological model for an accelerating universe (Rani et al., 2015) with a late-time acceleration phase for which the deceleration parameter $q \leq -1$ is established.

The very moment of the universe extinction may be likened either to a moment when the hologram animation is seized – like during the cinema show – by turning off the lights; or to a moment when an accelerated expansion of the universe with its spacetime coiling is terminated by the Big Rip in the black hole. While the former case implies primarily a state of the universal darkness the latter one implies primarily a state of the universal spacelessness-timelessness. Nevertheless, the thoughts-provoking idea of the end of time in understanding our universe (Barbour, 1999) may also imply an existence of a parallel universe in which time is passing in a countermarch (Boyle et al., 2022); but with the same course of the same events as they happen in our universe. Travel back in time would have then actually meant travel back to the future (Tobar & Costa, 2020). Which could explain a paradox of impossibility of making any change in any event that happened in the past so as to affect the presence – and simultaneously a possibility of knowing the future events. While seemingly never befallen yet potentially encountered events as alternatives to those which have been really experienced would have then passed by as an imaginary component of the complex cosmic game.

5. Reasoning About the Human Mind, the Universe, and God

An outlined interplay of the π-aided and e-aided scale factor variables effective in the equations proposed above as twin-like weavers of the geometrical field unity puzzle – as it would occupied Einstein's mind a century ago – is presented there by his imaginarily revived figure to deliberation. He has just realized that not a high complexity but per contra the utmost mathematical simplicity may solve such a puzzle. Which could be why he was unable to solve a century ago a fundamental enigma of the geometrical field unity of everything in the universe – and why his method of the thought experiment has been applied there by his own fictitious juvenile personage as his scientific legacy. On its grounds it may be argued that a controversy about cosmological models has originated paradoxically to a large extent from intricately sophisticated mathematical interpretations of observational data.

From now on juvenile Einstein keeps in his view the statements that *"...the general adherence to the mainstream concordance Λ-CDM model does not leave too much room for thinkers outside the accepted cosmic paradigm..."* (Martinez & Trimble, 2009); that *"...such model presently gives the best picture of how cosmic structures have emerged from the expanding background and have formed the cosmic web..."* (Vazza & Feletti, 2020); that the flat cosmological constant model remains the simplest and the best *"...to explain the current data..."* while giving *"...no reason to prefer any more complex model over the concordance cosmology, the flat cosmological constant..."* (Davis et al., 2007); that *"...nature represents its simplicity in various physical systems..."* (Nojiri et al., 2017); that it is the Lambert W function which *"...seems to be ubiquitous in nature..."* (Corless et al., 1996); that *"...transcendental reasoning in the spirit of Kant is becoming increasingly relevant in philosophy of science..."* (Hoffmann, 2019); that there have been outlined *"...transcendental perspectives on modern physics..."* (Bitbol et al., 2009); and what's more, that *"Transcendental arguments may be most important in physics."* as they were applied, for example, by Einstein for the creation of his theory of relativity which *"...abandons Euclidean geometry as an a priori chosen*

reference system for determining the spatial structure of the universe..." (Hoffmann, 2019); and last but not least that *"...fluids with logarithmic equation of state may lead to interesting cosmological behaviour..."* (Odintsov et al., 2018).

Having considered all of that and having surmised some divine will in the patterns of nature – and the essence of God in its transcendence – the young cosmic trail rider becomes truly aware why God cannot play dice with the universe; since the whole universe is nothing else but a mirrored face – or brain – of God. And herein – on grounds of findings about *"...the similarities between two of the most challenging and complex systems in Nature: the network of neuronal cells in the human brain, and the cosmic network of galaxies..."* which may be *"...shaped by similar principles of network dynamics, despite the radically different scales and processes at play."* (Vazza & Feletti, 2020) – young Einstein takes the order of magnitude 10^{27} as it has been reported by the authors for the scale difference between the cosmic/brain networks into account and wonders: "What if it could reflect a proportional difference in scales[7] between the size of the human brain and of the whole universe?" Since, when the human brain is viewed in its scale as a 'tiny universe' ($a_{0\ (human\ brain)} \approx 10^{-27}$ versus $a_{0\ (universe)} = 1$) its radius should be calculable according the equation proposed above. The answer $12.7736\ [cm]$ he gets corresponds to $8730.29\ [cm^3]$ which is more than a brain volume of the sperm whale; while an average brain volume in the middle-aged females has been reported to be $1115.76\ [cm^3]$ (Ritchie et al., 2018). Yet, with respect to cortical folding (Toro et al., 2008) it is more reliable to consider the surface area of the human cerebral cortex. Taking the reported $2400\ [cm^2]$ into account the calculated radius $12.7736\ [cm]$ corresponds then to the surface area $2050.39\ [cm^2]$; which fits quite well the fact that female brains are approximately 10% smaller than male brains. No matter how speculative these findings are they suggest the idea that in evolution of the galactic network structures there has been implicitly encoded evolution of the neuronal networks. Which makes juvenile Einstein declare: "God is not playing dice with the network dynamics; neither that of the galaxies nor that of the human brain."

Yet, the Universe itself cannot be a product of any Divine activity (or of any super intelligent simulation as it was suggested (Bostrom, 2003)) since in that case a position of God – or the programmer – would have been found 'outside' (Lane, 2021). And our presence would have been in that case a proof of our nonexistence – the finding which has emerged from an imaginary debate between Kublai Khan and Marco Polo about an existence of 'Invisible cities' (Calvino, 1974). Contrary to that there stands the idea of pandeism as *"...the belief that God chose to wholly become our Universe, imposing principles at this Becoming that have fostered the lawful evolution of multifarious structures, including life and consciousness."* According a particular form of pandeism, living God pandeism, *"... our Universe inherits all of God's unsurpassable attributes – reality, unity, consciousness, knowledge, intelligence, and effectiveness – and includes as much reality, conscious and unconscious, as is possible consistent with retaining those attributes. God and the Universe, together 'God-and-Universe,' is also eternal into the future and the past."* (Lane, 2021).

With respect to Kant's transcendental cosmology (Fugate, 2019) and with reference to the space-time unity and the mass-energy unity the idea of the 'God-and-Universe' unity seems to be evincible. Understanding this unity suggests that God, the Universe, and the human mind cannot be separated from each other – since they are the One. Actually, a journey towards understanding this unity represents a leitmotif of the allegorical tale 'The Conference of the Birds' by Farid al-Din 'Attar (2017) in which a hoopoe as a mastermind eggs other birds on to their struggle for an escape from bonds of an ephemeral earthiness so as to reach a sphere

of ethereality. Just from its perspective they may find out at last that a mystery of Divinity, Eternity, and Immortality has been ever hidden in themselves.

Conclusions

"Any event is like a ball decorated with entangled ornaments," argues Alexander Grin (1975) in his magical parable of the 'Binary Star Man' who was able to fly just by virtue of his own mental capacity. The first book edition of the tale entitled 'The Shining World' (Grin, 1924) was published two years before Albert Einstein declared in a letter sent on 4 December 1926 to Max Born (Einstein, 2005; Einstein et al., 2005) that God does not play dice with the universe: *"The theory says a lot, but does not really bring us any closer to the secret of the 'Old One.' I, at any rate, am convinced that He is not playing at dice."* In these two parables the novelist's and the physicist's thoughts met each other to express almost simultaneously similar understanding the nature of all the existence. *"Time and space were to him measures of the same eternity,"* quotes Margarit Tadevosyan (2005) words of Vladimir Nabokov (1992) about Grin's mind. In a way, Grin could sense that *'entangled ornaments'* which decorated and would decorate the universe did not originate from God playing at dice. Rather, their pattern could be encrypted in and attributed to an orchestrated action of certain numbers. As it has been illustrated by Einstein's imaginary cosmic adventure they could be represented by the numbers of Euler's identity that relates together two principal transcendental numbers, 0 and 1, with two principal digital numbers, and, and with the imaginary unit *i*: $e^{i\pi} + 1 = 0$. And since this imaginary unit may be whatever it means it might be identifiable herein just as the 'Old One', the 'Great Composer', the 'Great Conductor', the 'Great Animator'; the 'Supreme Being'; the 'Chief Referee'. Or simply God – who has according Einstein's original conviction never played dice with the Universe (Carr, 2004; Carter, 2006; Ursic, 2006; Musser, 2015; Lincoln, 2017).

Which calls for an anecdote about Leonhard Euler, Denis Diderot, and a mathematical proof of God's existence (Robertson, 1996). One day in 1773 both scholars met each other in St. Petersburg in the court of Catherine the Great: Czarina's highly privileged and respected mathematician, a staff member of the St. Petersburg Academy of Sciences, whom she had had back soon after her accession to the throne, and one of her lately invited scholars, a leading philosopher and chief encyclopaedist of the French Enlightenment. Yet, before long Czarina realized that even the candlestick of Enlightenment entailed its dark place: a gospel of the rationalistic atheism that harbingered an accession of the new 'God of Science'. With the idea that Diderot – as a person who turned out to be an overt denier of the existence of God – should be debunked, discredited, and silenced somehow, preferably by his own belief in science, she set for him an ingenious trap. Having reliance on Diderot's scientific eagerness Czarina sensed properly that a rumour released by her courtiers about a hot news that Euler had discovered a mathematical proof of God's existence was to reach his ears soon. Euler as a firm believer in God accepted Catherine's plot (one may deduce he enjoyed it with a certain portion of schadenfreude). And since Diderot really asked to join a social event at which the proof would be presented a scientific farce could be launched. During the event Euler solemnly proclaimed an equation (mostly reported as $[(a + b^n)/n] = x$ (Gillings, 1954)) as a scientific statement addressed directly to Diderot as a proof of God's existence. The philosopher was unable to formulate any witty counterargument by which the proof could be pronounced for its unreliable evidence invalid. Euler's language of 'irrational theism' gagged Diderot's language of 'rational atheism' – and soon afterwards the philosopher returned to France.

The anecdote authenticity was called into question in the past (Brown, 1942; Gillings, 1954; Struik, 1967) but a genuineness of Euler's gladsome, humorous, and charming character (Reiner & Reiner, 1990; Debnath, 2009; Calinger, 2016) supports its plausibility. Anyway, the question whether God can be really proved mathematically has remained open up to now (Robertson, 1996; Bischoff, 2022). But maybe there is an opportunity to be closed just by Einstein's fictitiously revived young figure whose trail ride around the expanding universe is going to be over. Euler's identity indicates that the rider has circumscribed on his Pegasus of Imagination but a semicircle of the universe so far: from +1 to -1. Fortunately, there is an algebraic trick: $e^{i\pi} = 1/e^{i\pi}$ (therefore $e^{i2\pi = 1}$; while $e^{i\pi} = -1$) which fuels his journey to advance in a specular symmetry of the same counterclockwise direction till bringing him back to the springing point. Just at the moment his imaginary journey taken under the indiscernible guidance of Euler's identity is ending in real terms. Then, could this identity harbor within the principles of symmetry and reciprocity a scheme of the geometric/algebraic field unity of the universe? With its 'contra-twin' parallel existence? And with a presence of God in all of the created things? And eventually, could the equation $[(a + b^n)/n] = x$ (as it is referred above as an enigmatic proof of God's existence) represent Euler's ingeniously encrypted notation of the identity? Imagine, $a = 1$; $b = e$; $n = i\pi$ yield $[(1 + e^{i\pi})/i\pi] = x$; which implies $x = 0/i\pi$, and hence $i \neq 0$. To conclude, i may mean 'Everything' except for 'Nothing'.

"That which is above is from that which is below, and that which is below is from that which is above...", an inscription from 'The Tabula Smaragdina' (Holmyard, 1923) hits off casually the heart of Euler's identity. The juvenile adventurer identifies its imaginary unit as a supreme authority that controls from its unknowable and indefinable position beyond one's reach an orchestrated action of the remaining four numbers; of which e and π relate to spacetime while zero and unity reflect Leibniz' concept of nothingness-infiniteness as two cut-off points of the evolution of galactic networks and of neural networks of the human brain. On grounds of ascertainments that: *"In nature everything happens by degrees, and nothing by jumps."* (Leibniz, 1996); *"[...but] continuity is only a mathematical technique for approximating very finely grained things. The world is subtly discrete, not continuous."* (Rovelli, 2018a) the universe evolution is found by young Einstein to be a highly sophisticated discrete yet continuous game. Being principally different from playing at dice or from any card game it is viewed by him as a collective mass-energy match given on the space-time playground. With its expansion the match is running into the countless variants of physically more and more disordered schemes characterized by the entropy growth – that *"...distinguishes the past from the future for us and leads to the unfolding of the cosmos."* (Rovelli, 2018b). Which points out that the scale of the space-time playground represents a measure of entropy of the performed mass-energy game – and of an arrow of time defining its direction. And since Einstein's imaginary journey demonstrated on one hand that an accelerated expansion of the playground implied an accelerated dilation of time reserved for the match – and on the other that there was established an upper bound for the playground scale – the maximum possible entropy of the performed cosmic game has to exist then as well. When it is met both an arrow of time and time itself along with the imaginary unit vanish away and the Universe with all of its ever existing forms and events, thoughts and memories, unveils its all-encompassing face of Divinity, Eternity, and Immortality attributable to God.

Notes

[1] Roots of the function have been iterated as follows: $log(x_1) \approx 5.5545318206$, i.e. $x_1 \approx 358535.21706$, and $log(x_2) \approx -0.53640346864$, i.e., $x_2 \approx 0.29080142568$). The function has a local minimum equal to $e/3$ for $x = e^3$.

[2] Graph 4.4.2., an open source application for plotting mathematical functions (the GNU General Public License) was applied to delineate graphs shown in Fig. 2, Fig. 3, and Fig. 5.

[3] The constant is definable as $D_c = ([1/[2(e^{\pi}+1)]] + \pi)^2 \cdot 10^6 \approx 10.00017019 \cdot 10^6$ $[km^2 \cdot s^{-2}) \cdot Mpc^{-2}]$. One may note that $[1/[2(e^{\pi}+1)]] + \pi) \approx 3.16230457$ $[km \cdot s^{-1} \cdot Mpc^{-1}]$ is close to $3.16227189\ldots$ $[km \cdot s^{-1} \cdot Mpc^{-1}]$: defined by the Hubble parameter equation for $a = 3.62$; and numerically also close to $\sqrt{10} \approx 3.16227766$:) as well as to $[10(10 + \pi^2)/(2\pi\sqrt{10})]^{1/2} \approx 3.16231171$:. And since $10(10 + \pi^2)/(2\pi\sqrt{10}) \approx 10.00021534$ implies $(10 + \pi^2)/(2\pi\sqrt{10}) \approx 1.000021534$ then $(10 + \pi^2)/(2\pi\sqrt{10}) \approx ([1/[2(e^{\pi}+1)]]+\pi)^2/10$. Should all of this point to a special role of the number ten in cosmology is left to discussion.

[4] The deceleration parameter calculated for a present epoch $q_0 \approx -0.499$ nears to the referential $q_0 \approx -0.5$. And the Hubble parameter calculated for a present epoch $H_0 \approx 72.42$ $[km \cdot s^{-1} \cdot Mpc^{-1}]$ lies between an upper limit of H_0 similar to 71 $[km \cdot s^{-1} \cdot Mpc^{-1}]$ yielded by early universe physics and H_0 similar to 73 $[km \cdot s^{-1} \cdot Mpc^{-1}]$ to which various local determinations may be converging (Krishnan et al., 2021).

[5] Proposals of the Lambert W function-based equations for the deceleration parameter and for the energy density are presented there; along with a disclosure of their authors that *"... due to a high degree of complexity in the expression for ρ, we are unable to solve it for the scale factor..."*: $q = (3/2)\{1 + \vartheta_1\, ln[W(a)] + \vartheta_2\, W(a)^3\}$-1; $\rho = \rho_0\, exp[-3\{ln[W(a)][\vartheta_1\, W(a) + \vartheta_1 + 1]\, W(a)(1 - \vartheta_1) + (\vartheta_2)/12)\, W(a)^3\, [4 + 3W(a)]\}]$.

[6] The universe radius $r \approx 3.8657 \cdot 10^{-35}$ $[m]$ calculated for the scale factor $a = 10^{-35}$ edges the Planck length: $l_p \approx 1.61625 \cdot 10\text{-}35$ $[m]$ while the age $A \approx 1.2895 \cdot 10^{-43}$ $[s]$ edges the Planck time $t_p \approx 5.39125 \cdot 10^{-44}$ $[s]$. Both data calculated for a present epoch ($a_0 = 1$): $r \approx 1.2773 \cdot$ $[km]$ and $A \approx 13.502 \cdot 10^9$ $[years]$ are 2% under current official data ($r \approx 1.3043 \cdot 10^{23}$ $[km]$ and $A \approx 13.787 \cdot 10^9$ $[years]$). For the maximum scale factor ($a_{max} = e^{\pi}+1$) $r_{max} \approx 3.1079 \cdot 10^{36}$ $[km]$ and $A_{max} \approx 3.2852 \cdot 10^{23}$ $[years]$ were calculated.

[7] Actually, the scaling ratio of $1.875 \cdot 10^{27}$ between the cosmic web and the human cerebral and cerebellar cortex when their power spectra were examined (*1,6 mm* in brain samples corresponded to 100 *Mpc* in the cosmic web) is referred. Synthetic samples of the cosmic web from a high-resolution (2400^3 cells and dark matter particles) simulation of a cubic 100^3 $[Mpc^3]$ cosmic volume were used.

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Figures

Fig. 1: Albert Einstein in a Munich photo studio (1894). Unknown author. Image by: Wikimedia Commons, the free media repository. Public domain. https://commons.wikimedia.org/wiki/File:Albert_Einstein_as_a_child.jpg

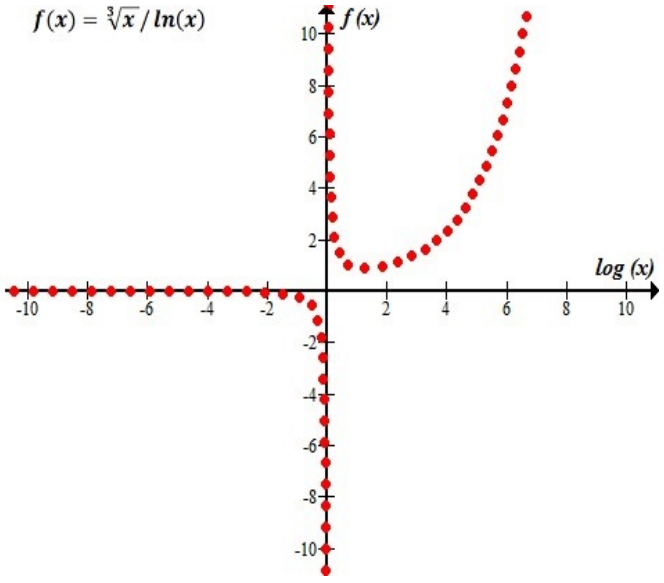

Fig. 2: The graph of a logarithmic-exponential discontinuous function plotted against. The function is derived from a transcendental equation.

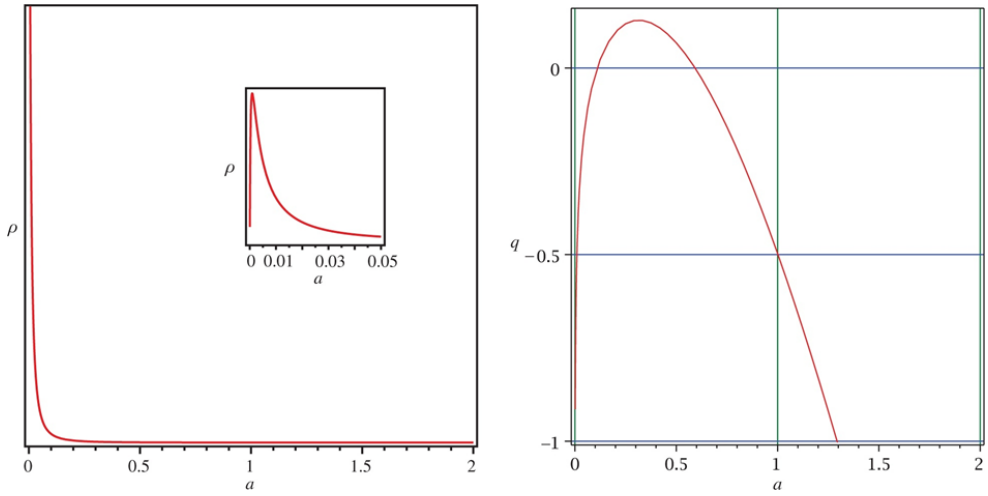

Fig. 3: Behaviour of the energy density and of the deceleration parameter as a function of the scale factor according the Lambert W function-based equations presented in: Saha, S. & Bamba, K.: "The Lambert W Function: A Newcomer in the Cosmology Class?"; in: Holthaus, M., Fetecau, C. & Kiefer, C. (eds.), Zeitschrift für Naturforschung A – Physical Sciences, 75(1), Berlin/Boston: Walter de Gruyter GmbH, 2020, pp. 23-27, fig. 1. Reprinted with a permission from the publisher.

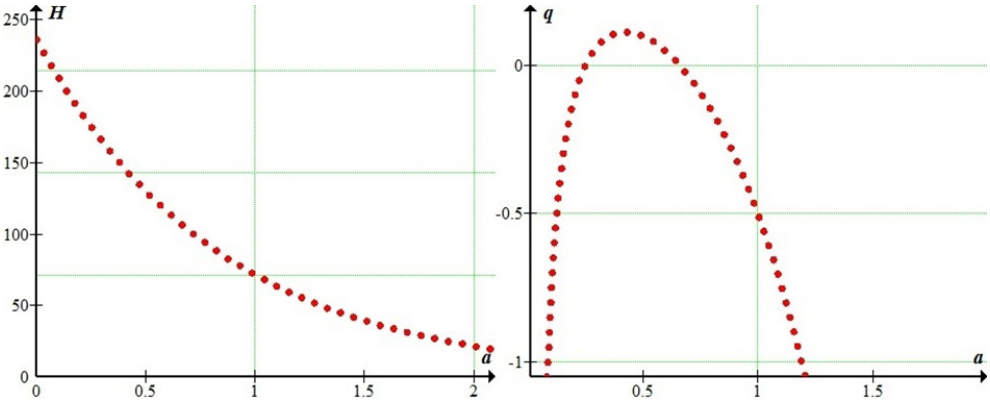

Fig. 4: Behaviour of the Hubble parameter and of the deceleration parameter as a function of the scale factor according the logarithmic-exponential discontinuous function ()-based equations presented in this paper.

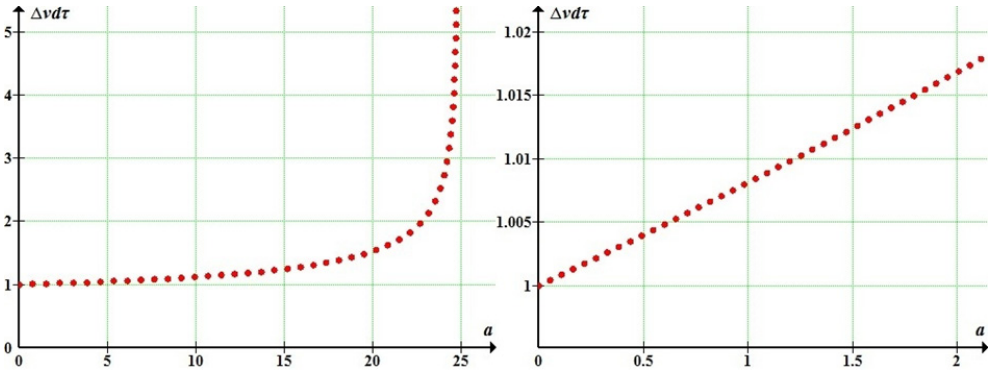

Fig. 5: Behaviour of the time dilation factor as a function of the scale factor according the equation presented in this paper.

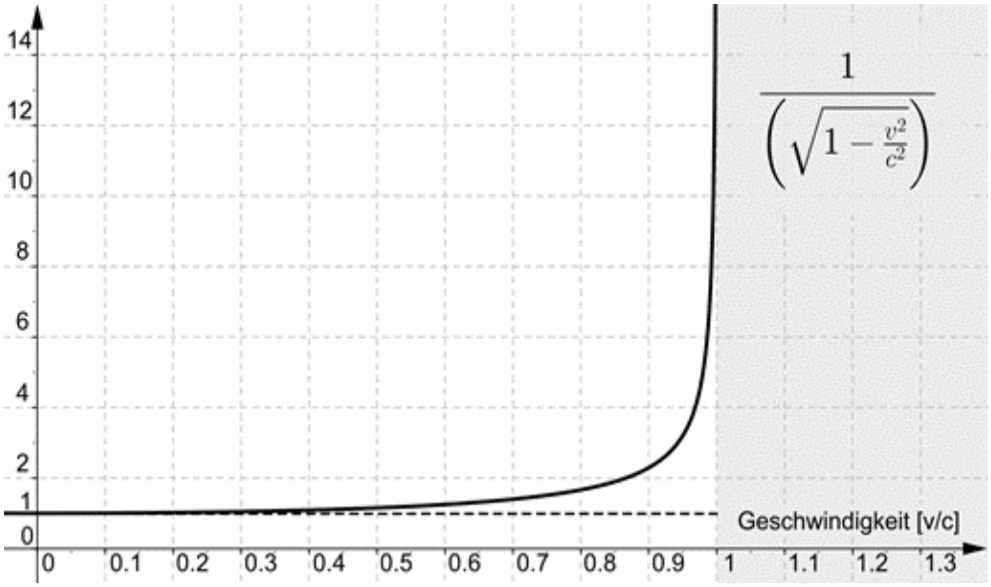

Fig. 6: Behaviour of the Lorentz factor () as a function of the relative velocity (). Figure from: File:Lorentzfaktor.svg: Klamannderivative work: MikeRun, Public domain, via Wikimedia Commons. https://commons.wikimedia.org/wiki/File:Lorentz-factor.svg