Cosmological Ideas of Johannes Kepler in Their Relation to Antiquity Based on the Treatise *Harmonices Mundi*

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The article presents the results of the authors’ research on the cosmological views of Johannes Kepler in his Latin treatise *Harmonices Mundi*. The authors discuss the background of composing the treatise concerning the philosophical, religious, and cultural contexts of that period, and give a brief overview of the structure of the treatise, Kepler’s theory and his third law of harmony. The paper demonstrates the connection and reception of philosophical ideas on the harmony of the world in Kepler’s treatise and ancient philosophers, such as Pythagoras, Plato, and Proclus. This shows that Kepler’s philosophical and scientific views largely depended on his theological background. The authors discuss the perception of ancient cosmological ideas in Kepler’s *Harmonices Mundi* relating to Pythagorean tetractys, five Platonic solids or regular polyhedral, and cosmic harmony as well as Proclus’ theoretical mathematics.

Keywords: Kepler, *Harmonices Mundi*, cosmology, philosophy, harmony, universe, mathematics, astronomy.

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Introduction

The Latin treatise *Harmonices Mundi* is a profound and intellectual study written by Johannes Kepler and published in 1619. It represents a continuation and expansion of his earlier work on planetary motion, based upon the laws that he formulated in his *Astronomia Nova* and *Epitome of Copernican Astronomy*. In *Harmonices Mundi*, Kepler explores the concept of harmony in the universe, seeking to find a connection between the motion of the planets and the principles of music and geometry. Inspired by the ancient Greek idea of the “music of the spheres” attributed to Pythagoras, Kepler sought to find musical harmonies in the motions of celestial bodies. The ancient belief proposed that the celestial spheres, each carrying a planet, emitted musical tones as they moved, resulting in cosmic symphony.

It was in Kepler’s *Harmonices Mundi* that his third law was formulated, also known as the “law of harmonies.” This law, which he had derived earlier, states that the square of a planet’s orbital period is directly proportional to the cube of its average distance from the Sun. Kepler believed that this mathematical relationship had a deeper significance and saw it as a reflection of the harmonies present in music. In his treatise, Kepler explored the relationships between planetary distances and their orbital periods. He found fascinating numerical patterns and ratios between the planets’ distances, which he interpreted as musical intervals, akin to those found in the tuning of musical instruments. Kepler delved into the relationship between geometry and harmony. He believed that the geometrical arrangement of the planetary orbits and the five regular polyhedra (tetrahedron, hexahedron, octahedron, dodecahedron, and icosahedron) played a crucial role in determining the harmony of the cosmos. He associated these shapes with the distances of the planets from the Sun. Although some of Kepler’s ideas about harmony and the geometry of the universe may not align with modern scientific understanding, his treatise laid the groundwork for future explorations in the fields of astronomy, cosmology, mathematics, and music.

The objectives of our study are the following: 1) to describe the philosophical, cultural, and religious context of composing the treatise; 2) to trace the idea of world harmony from antiquity to Kepler’s time; 3) to analyze the idiosyncrasies of Kepler’s perception of ancient ideas.

The methodology of the present paper is based on a number of approaches to the study of Kepler’s intellectual heritage: a) philosophical, represented by G.Claessens (2011), C.A. Cardona (2016), J.Michalik (2019), and O.Bazaluk (2021); b) historical, represented by J. Godwin (1993), J.R.Voelkel (1999) and P.Gouk (2008); c) astronomical, represented by J.V. Field (1982; 1988), B.Gingras (2003), and P.J.Boner (2013).

Philosophical, Cultural, and Religious Contexts of Composing the Treatise *Harmonices Mundi*

Johannes Kepler’s treatise *Harmonices Mundi* was written in 1619, almost twenty-five years after he graduated from the Swabian University of Tübingen, where he mainly studied philosophy and theology and also attended lectures in astronomy and mathematics. Scholasticism dominated the educational system of Kepler’s time and was aimed at teaching Christian truths through philosophy and theology, while also based on the physical ideas of Aristotle. Kepler’s theological education contributed to his worldview and scientific research. He was aware of Plato’s idea of the Creator, but substituted it with the Christian God. Furthermore, in Book IV of *Harmonices Mundi* he claimed that Plato’s dialogue in *Timaeus* could be treated by an attentive reader as a commentary on Genesis or the first book
of Moses, which were converted to the Pythagorean philosophy.\(^1\) In general, scholars agree that Kepler’s idea of religion was based on rationality and pertained to God, humanity, and nature.\(^2\)

The early seventeenth century was a time when Copernican theory was considered absurd, and students were obliged to study astronomy according to Ptolemy’s theory. Through his teacher, Michael Maestlin, Kepler became acquainted with and accepted the heliocentric theory, which contributed greatly to his subsequent scientific works in the future. Scholars admit that it was Kepler’s achievement that he substituted a dynamic system for the formal schemes, of the natural law for mathematical rule and gave a causal explanation for the mathematical description of motion.\(^3\) At the same time, people started raising questions about the world order and what holds its elements together; in this regard, they were affected by Platonism. Plato’s philosophy in late antiquity was developed into Neo-Platonism (mathematical philosophy) and Neo-Pythagoreanism (mathematics of natural philosophy). Below, we discuss the reception of these ideas in Kepler’s treatise.

The harmonic principles of world order, which were discussed in Kepler’s treatise, were found by him in *Harmonics* by Ptolemy. Though based on geocentric astronomy, this book was treated fairly by Kepler for pioneering the dissemination of the study of the harmonious structure of the universe. In Kepler’s time, six planets had been discovered, and he gave the explanations for this number and found harmonic proportions in their motion. His treatise consists of five books. The first two books are devoted to geometry: Book One is mainly based on Euclid, while the second one is marked by originality. The third book analyses music harmony along with recent astronomical discoveries. The fourth book deals with astrology, and the fifth book concerns astronomy and harmonies. The main principles of Kepler’s theory could be described as follows: 1) harmonies exist but they do not produce any sound; 2) harmonies are perceived from the Sun (heliocentric view); 3) harmonies are polyphonic; 4) harmonies follow the proportions of just intonation.\(^4\)

It should also be mentioned that harmony was understood as the law of the world’s organisation rather than only a musical term. The idea of harmony can be traced back to antiquity and was first attributed to Pythagoras and his followers. This idea was based on the presumption that the circular motion of the spheres produced a sound. People do not hear this sound because the spheres are present with them since their birth, so they do not perceive this sound, being unable to experience the absence of the spheres. This explanation was rejected by Aristotle, who said that we hear no sound because there is no sound. Along with the term “harmonics” was used another, “canonics,” associated with Euclid’s work *Sectio Canonis*, in which the author explained harmonics with a mathematical theorem.

The tradition of studying musical harmony along with mathematics, geometry, and astronomy goes back to antiquity, and due to Boethius, these subjects comprised the system of the quadrivium, analogous to the trivium, which comprised rhetoric, logic, and grammar. This system was part of the educational curriculum from the twelfth to the eighteenth centuries. It was also Boethius who proposed the classification of music into three types: *musica instrumentalis* (instrumental music as the acoustic harmony), *musica humana* (human music as the harmony of soul and body), and *musica mundana* (music of

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\(^1\) (Kepler, *Harmonice Mundi* IV).

\(^2\) (Kozhamthadam, 1994).

\(^3\) (Caspar, 1993).

\(^4\) (Walker, 1978).
universal harmony). The last type of this classification is represented by philosophers who can perceive and judge the world’s harmony. When the new texts became available for the majority of readers, the view on the world’s harmony changed. By these texts, we mean the published editions of *Corpus Hermeticum*, ascribed to Hermes Trismegistus, in 1463, Plato’s *Timaeus* and *Republic* in 1484, and *Enneads* by Plotinus in 1492. These writings contributed to the spreading of a new understanding of harmonies, and knowledge of alchemy, astrology, and magic arts.

The idea of the harmony of the world is not taken seriously nowadays, but for Kepler and his contemporaries, it was an issue of serious philosophical, mathematical, and astronomical consideration. Almost simultaneously, several treatises on harmony appeared in different countries and different languages: *Harmonices Mundi* by Kepler (in 1619), *Harmonie Universelle* by Mersenne (in 1636–1637), and *Musurgia Universalis* by Kircher (in 1650). In his treatise, Kepler distinguished between the two harmonies, sensible and genuine, stating that sensible harmony relates to sensible things and may be of different kinds, as these things are different, whereas genuine harmony does not relate to sensible things and remains one and the same kind. He also describes in details the elements of the sensible harmony, which consists of two sensible things of the same kind; the soul, which compares these two things; reception of these things and necessary proportion, which is the harmony itself.

### Kepler and the Pythagoreans

In Greek philosophy, the idea of a harmonious world was first observed by Pythagoras and his followers, who identified the intervals of the fourth, fifth, and octave with their ratios. These ideas, which were formulated at the end of the fifth century B.C., were preserved orally until they were recorded by Aristotle, Plato, and others. The Pythagoreans recognised seven planets: Mercury, Venus, Mars, Jupiter, Saturn, the Moon, and the Sun. They believed that each planet produced a particular note, and their pitch was determined by velocity, which depended on the distance from the Earth. This idea was supported by Kepler as well. It is worth mentioning that Scotus Eriugena, who lived in the ninth century A.D., espoused this theory. According to Eriugena, the scholars might have explained pitch change when the planets’ distance from Earth changed accordingly. Pythagoreans distinguished between three main consonances as the octave, fourth, and fifth, with their numeric relationships as 4:3; 3:2 and 2:1. This idea was visually demonstrated in a triangle with ten spots known as Pythagorean tetractys. Aristotle summarises this Pythagorean idea of numbers, stating that everything in nature is based on numeric organisation, and the elements of numbers are the elements of all things, and the heavenly harmony is a number.

So, according to the Pythagoreans, harmony depended on simple numeric ratios, and any other random combination of sounds could not produce pleasant sounds. However, this restriction to the consonances resulted in not including the thirds and sixths, which did not allow polyphony. Kepler commented on Pythagorean ideas in a way that they were so much engaged in philosophising in numbers that they started judging a melody, as well as consonants and dissonants, not with their ears, but with numbers. Kepler’s view was that we should find the cause of the consonance in geometry, rather than in number. He gave several reasons for this in his treatise: 1) theological reason argued that there is no explanation for...
God’s choice of certain numbers in producing consonance. According to Kepler, God took the notes and intervals and arranged them in the shape of human spirits. This idea was inspired by God’s contemplation of numbers, though it was not clear why certain numbers conformed with musical instruments and other numbers did not conform with them; 9) rational reason assumed that musical intervals are continuous and cannot be rooted in discrete numbers. He distinguished between consonant and dissonant intervals, stating that consonant intervals are harmonious and thus have an intellectual basis, while dissonant intervals cannot be known as they do not enter the mind; 10) philosophical reason explained that numbers are inferior to geometrical figures in the epistemological and metaphysical sense. 11

Another Pythagorean idea, which is employed in Kepler’s work and is present in illustrations to Harmonices Mundi, was the notion of five regular solids or five regular polyhedra, which are sometimes called the five Platonic solids: the tetrahedron, hexahedron, octahedron, dodecahedron, and icosahedron. Proclus ascribed this idea to the Pythagoreans, and Plato associated these solids with four physical elements as follows: Earth with the hexahedron, water with the icosahedron, fire with the tetrahedron, air with the octahedron, and the dodecahedron was associated with Heaven. This idea was shown in Kepler’s drawings in the second book of his treatise (Kepler, 1997). In Plato’s Timaeus, these solids consisted of equilateral triangles, but Kepler discovered two more new regular polyhedra, star octagons, and star decagons, which involve twelve-star pentagons and two semisolid congruences. 12

Kepler applied the five Platonic solids to explain the number of the six known planets, stating that these solids delimit the distances between the planets. He placed a certain solid between each planet in the following order: a hexahedron was placed between Saturn and Jupiter, a tetrahedron between Jupiter and Mars, a dodecahedron between Mars and Earth, an icosahedron between Earth and Venus, and an octahedron between Mercury and Venus.

**Kepler and Platonic ideas**

The idea of harmony in Plato’s compositions is mentioned twice, in his Republic and in Timaeus. Let us consider these two points in detail. In the tenth book of Republic is a passage relating to the eight heavenly spheres, which are described as circles, revolving around with different speed. There is a siren on each of the rims of these circles, who is singing a particular musical tone, producing harmonious sound together. 13 Regarding this passage, it is worth noting that the image of a siren in Pythagorean view differs from Homer’s poetic description of these creatures in The Odyssey, as they produce heavenly music, rather than bringing death to sailors. Another thing concerns the word “Ανάγκης,” which is translated as “necessity,” so the motion of the spheres is turning on the knees of “necessity.” In Aristotle’s Metaphysics we find the definition of “necessity” as something that is the condition of being. 14

Another passage from Plato concerning the harmonies of the world is found in Timaeus. There, we read that the Creator made a soul out of three elements: he took the indivisible and divisible essences, combined them, and created the third essence, placing it between the first two. This essence pertained to the nature of divisible and indivisible essences. Then the

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9 Ibid.
10 Ibid.
11 (Kepler, Harmonice Mundi Apolog.)
12 (Kepler, Harmonice Mundi II).
13 (Plato, Republic 617a).
14 (Aristotle, Metaphysics 1015a).
Creator mingled the three essences and out of them made one, which was again divided into the number of parts that was fitting, and each part became a constituent of the three elements of the soul: Sameness, Difference, and Existence. The Creator’s division was made in the following way, resulting in certain numbers (1, 2, 4, 8, and 1, 3, 9, 27).  

Kepler rejects Plato’s numerology and gives priority to geometrical objects. In one of Kepler’s letters, we read that Ptolemy had the courage to think of the world’s creation in terms of geometry and Euclid’s philosophy, because he did not know about God. According to his views, by geometrical objects, we can explain why God created this world in this way and not otherwise. When Kepler quoted the passage about harmonic proportions from *Timaeus* in his treatise *Harmonices Mundi*, he mentioned that Aristotle did not support, and even rejected, this idea, but he himself thought that these words from *Timaeus* should not have been taken literally. Then he describes another idea from *Timaeus* concerning the role of the mind in distinguishing proportions, and seemingly shared it with the philosopher. The idea was that the mind rather than the senses is responsible for distinguishing between consonant proportions and dissonant ones, and it also distinguishes proportions from that which is proportioned.

### Kepler and Neo-Platonic Ideas

In his treatise *Harmonices Mundi* Kepler quotes from Proclus’ *Commentary on the Elements of Geometry*. It can be assumed that Kepler was acquainted with original Greek editions of Proclus in Plato’s *Opera Omnia*, which was published in 1534, and in Euclid, published in 1573, as these editions were available in the library of the University of Tübingen. The quotations and references to Proclus are present on the title pages of three of the books making up the treatise. In the fourth book, we can find the three-page-long Latin translation from the *Commentary on the First Book of Euclid’s Elements*. In the prooemium to the first book of *Harmonices Mundi*, Kepler called Proclus a theoretical philosopher who established the general principles of the whole essence of mathematics and distinguished between the limit as the form and the unlimited as the matter of geometrical objects.

As a theoretical philosopher of mathematics, Proclus stated that mathematical notions take place between Plato’s ideas and the objects of the sensible world. These objects of the sensible world belong to opinion, the ideas belong to the mind, and the mathematical notions are found between these two and belong to understanding. This is due to our imagination that mathematical concepts such as geometrical objects become visible and extended.

For Proclus, each geometrical object is ontologically and epistemologically equal, unlike Kepler. According to Kepler’s theory, priority is given to the circle and the arc, so he considered the other geometrical objects as derivative. The circle is the form of the soul, and the arc is a constituent part of the circle. Harmonization is created by these two objects: circle and arc. This leads to the central point of Kepler’s theory: God, who created this world and universe, made the geometrical elements the entities of our soul and established the circle as the basic source of geometrical objects. The other geometrical objects appear on the grounds of a comparison between the circle and the arc.

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16 (Kepler, 1951).
17 (Kepler, *Harmonice Mundi* III).
18 (Kepler, *Harmonice Mundi* Intro.).
19 (Proclus, 1970).
20 (Kepler, *Harmonice Mundi* IV).
Conclusions

Thus, Kepler’s *Harmonices Mundi* is a significant work of scientific and philosophical literature that explores the fascinating connections between planetary motion, musical harmony, and geometric order in the cosmos. It represents an ambitious attempt to find unity and harmony in the diverse workings of the universe, demonstrating Kepler’s relentless quest to understand the divine order behind the celestial phenomena.

Johannes Kepler, the influential astronomer and mathematician of the seventeenth century, had a deep appreciation for ancient Greek philosophy. He studied and drew inspiration from the works of several Greek thinkers, incorporating their ideas into his own scientific and cosmological investigations. Three Greek philosophers, in particular, had a significant influence on Kepler’s thought: Pythagoras, Plato, and Proclus.

Kepler was fascinated by the Pythagorean ideas of the “music of the spheres,” which suggested that the planets and celestial bodies emitted musical tones as they moved through the heavens. This concept fascinated Kepler, and he sought to find connections between planetary motion and musical harmony in his treatise *Harmonices Mundi*. Pythagorean numerology and the emphasis on geometric harmony also influenced Kepler’s views on the structure of the universe. Kepler’s belief in the significance of the five regular polyhedra (the Platonic solids) in organizing the spacing of the planetary orbits in *Harmonices Mundi* was rooted in the Pythagorean tradition of associating geometric shapes with cosmological principles. Unlike the Pythagoreans, Kepler recognized the thirds and sixths as consonants and added these intervals to the geometrical theory.

Kepler was deeply influenced by Platonic ideas, particularly the notion that the universe was an intelligible and mathematically ordered entity. Plato’s theory of forms posited that the physical world is a reflection of higher, ideal forms or archetypes. Kepler was drawn to this idea and believed that the principles governing planetary motion were rooted in a higher, divine order. Plato’s influence can also be seen in Kepler’s belief in the significance of mathematics as a tool for understanding the cosmos. Like Plato, Kepler thought that mathematics held the key to unlocking the secrets of the universe. He searched for mathematical patterns and harmonies in planetary motion, believing that they revealed the divine plan of the cosmos.

Kepler concurred with Proclus that the circle and the arc constitute the essence of the soul, but did not agree with Proclus about the other mathematical concepts, which were also attributed to the soul.

Overall, Kepler’s engagement with Greek philosophy, particularly the ideas of Pythagoras and Plato, profoundly shaped his approach to astronomy and cosmology. Their emphasis on mathematics, harmony, and the search for higher truths resonated with Kepler’s quest to understand the underlying order of the universe, leading him to make groundbreaking discoveries in the field of astronomy and lay the foundations for modern celestial mechanics.

References


